IMAC TUTORIAL

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Rates of Returns

- *Simple interest* assumes that the **same interest is earned per** day for the entire holding period.

- *Compound interest* assumes that the **principal and interest are received at the end of each compounding period, and then reinvested at the same rate**. The return compounds as many times as there are compounding periods in a holding period.

- Application of *continuous compounding* treats the return as if it accrued continuously, as a constant rate of growth on the initial principal plus all accrued interest.

Nominal vs Annual

- How do we compare two instruments with different payment frequencies? Are you better off paying 5% on an annual basis or 4.5% on a monthly basis?

- Convert the nominal rate into an annualized rate:

$$R_{\text{Annual}} = \left(1 + \frac{R_{\text{Nominal}}}{n}\right)^{n} - 1$$

where *n* is the number of periods per year for the instrument.

Question 1.

The nominal monthly rate for a loan is quoted at 5%.

- What is the equivalent annual rate?
- Semiannual rate?
- Continuous rate?

- *Hint: use*

$$R_{\text{Annual}} = \left(1 + \frac{R_{\text{Nominal}}}{n}\right)^{n} - 1$$

Question 1.

The nominal monthly rate for a loan is quoted at 5%.

- What is the equivalent annual rate?
- Semiannual rate?
- Continuous rate?

- *Hint: use*

$$R_{\text{Annual}} = \left(1 + \frac{R_{\text{Nominal}}}{n}\right)^{n} - 1$$

- Annual rate = 5.12%; semiannual rate = 5.05%; continuous rate = 4.99%.

Rates of Return

- Single Period HPR:

$$HPR = \frac{(P_1 + D_1 - P_0)}{P_0} = r_1$$

- Multiple holding periods:
  - Value (Dollar)-weighted returns:
    - Internal rate of return (IRR) considering the cash flow from or to an investment;
    - IRR which equates the FV of cashflows (inflows– outflows) from all of the individual periods plus the initial value of the portfolio to the portfolio's ending value;
    - depends on the timing of inflows and outflows of cash into the portfolio.
  - Time-weighted returns:
    - measures the compounded rate of growth of the portfolio's value during the period;
    - also referred to as geometric return as it is computed by taking the geometric average of the individual period returns;
    - does not depend on the timing of cash flows.

Rates of Return

- Time-weighted returns r_{TW}:

$$\left(1+r_{TW}\right)^n = (1+r_1)(1+r_2)\cdots(1+r_n)$$

$$r_{TW} = \left[(1+r_1)(1+r_2)\cdots(1+r_n)\right]^{1/n} - 1$$

Time Weighted

- Consider a **two-period** investment.

- ***Time-Weighted Return***: $\boldsymbol{r_{tw}}$

$1+r_1$ = end of year 1 value / (beginning of year 1 value)

$1+r_2$ = end of year 2 value / (beginning of year 2 value)
= end yr.2 value / (initial yr.2 value + turn-of-year contrib.)

$$r_{tw} = \sqrt{(1+r_1)\times(1+r_2)} - 1$$

Rates of Return

- Value (Dollar)-weighted returns r_{vw}

- For an investment over n (holding) periods

$$Ending\ value = \sum_{i=1}^{n-1} C_i (1 + r_{VW})^{n-i} + IV(1 + r_{VW})^n$$

where C_i = *amount invested in period i, taking into account contributions and withdrawals*

IV = *Initial value of investment*

Solve for r_{vw}.

Value (Dollar) Weighted

- Consider a **two-period** investment.

- ***Value-Weighted Return***: r_{vw}

  Ending Value = (Initial Value Invested)$(1+r_{vw})^2$ + (Contribution)$(1+r_{vw})$

- Although the Value-Weighted Return is what matters to the investor, the Time-Weighted Return is the best measure of an investment manager's performance.

Time & Dollar Weighted Returns

Question 2.

Consider the following data. Assume that contributions are made at the beginning of the year.

Year	Year-End Value	Contributions
1	$22.0	-
2	$20.1	-
3	$32.0	$5.0

(a) What is the time-weighted rate of return on the fund?
(b) What is the value (dollar) weighted rate of return on the fund?

Answer:
(a)R_{tw} = 7.93%
(b)R_{vw}= 9.78%

Time & Dollar Weighted Returns

1. **Dollar-Weighted Returns** (DWR)

- Equivalent to the internal rate of return (IRR) used in financial calculations
- The IRR measures the actual return earned on a beginning portfolio value and on any net contributions made during the period.
- The DWR equates all cash flows, including ending market value, with the beginning market value of the portfolio.
- Because the DWR is affected by cash flows to the portfolio, it measures the rate of return to the portfolio owner. However, because the DRW is heavily affected by cash flows, it is inappropriate to use when making comparisons to other portfolios or to market indexes.

2. **Time-Weighted Returns** (TWR)

- TWR typically is calculated for comparative purposes when cash flows occur between the beginning and the end of a period. TWRs are **unaffected by any cash flows to the portfolio**; therefore, they measure the actual rate of return earned by the portfolio manager.
- A Time-Weighted Rate of Return is a series of geometrically linked Internal Rates of Return.

Time & Dollar Weighted Returns

The dollar- weighted return and the time-weighted return, can produce different results, and at times these differences are substantial. The time-weighted return captures the rate of return actually earned by the portfolio manager, while the dollar-weighted return captures the rate of return earned by the portfolio owner.

DWR is time and dollar weighted, whereas the TWR is time weighted only.

For evaluating the performance of the portfolio manager, the time-weighted return should be used as the manager has no control over the deposits and withdrawals made by the clients. The objective is to measure the performance of the portfolio manager independent of the actions of the client, and this is better accomplished by using the time-weighted return.

Question 3.

You have the following data on a mutual fund:

Year	Beginning of Year Contributions	Year End Contributions	Year End Portfolio Value
1	0	6	106
2	5	8	148
3	4	7	167

Assume that year-end portfolio value is measured after the year-end contributions but before any contributions made in the beginning of the next year.

(a) Compute the time-weighted return on the fund from the end of year 1 through the end of year 3.

(b) Compute the dollar-weighted return on the fund over this period.

Distribution Statistics

- The property of a random variable is described by its probability distribution function.

- Descriptive statistics describe the essential characteristics of a distribution function without resorting to a full description of the entire probability distribution.

  – ***expected value or mean*** - center of mass of a distribution
  – ***variance and standard deviation*** - spread about the mean
  – ***skewnes***s - measure of symmetry about the mean
  – ***kurtosis*** - measure of tail thickness
    - ✓ skewness and kurtosis measure the "shape" independent of the mean and variance
      - ✓ Every **normal distribution** has
        – skewness coefficient = 0
        – kurtosis = 3

Risk/Return Measurement

- Modern Portfolio Theory defines **risk as the variability, or volatility, of returns.**
- **Volatility is often measured by the variance or the standard deviation**.
- Both the variance and the standard deviation measure the dispersion (or deviation) of return from its average over a defined period.
- The variance and the standard deviation measure **total risk of returns** of an asset or portfolio.
- Beta measures the market, or systematic, risk associated with portfolio returns. It represents the risk that cannot be diversified away.
- **Standard deviation: always positive**.
- **Beta: may be positive or negative.**

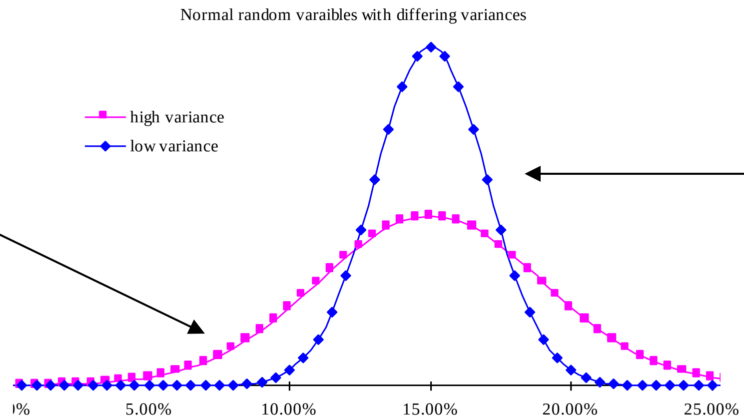
Risk Measurement

- Asset returns are typically modeled as "random" variables. We don't expect them to be the same every day, but on average we expect a particular return.

- To examine how "random" a stock's return is we use variance or standard deviation. This variance measures how the returns are dispersed around the mean.

Stock

High Variance/
High Standard Deviation

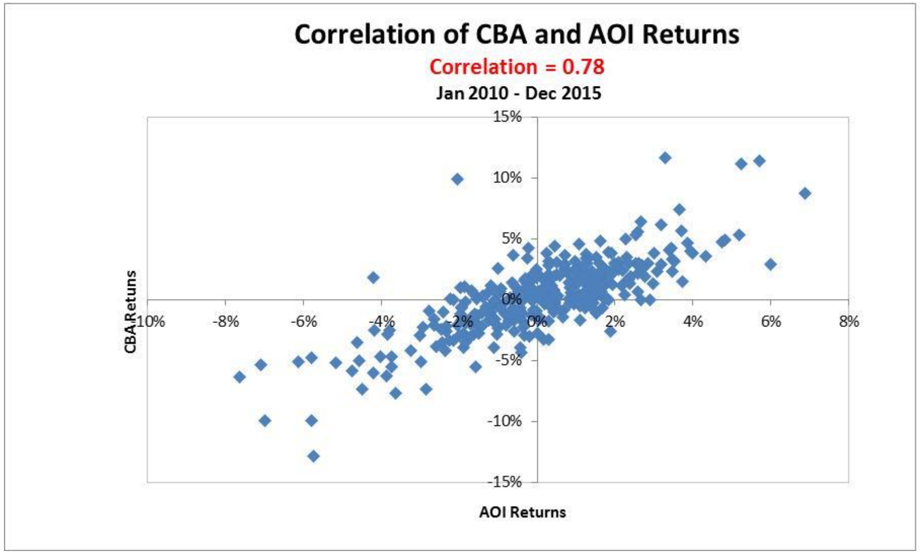


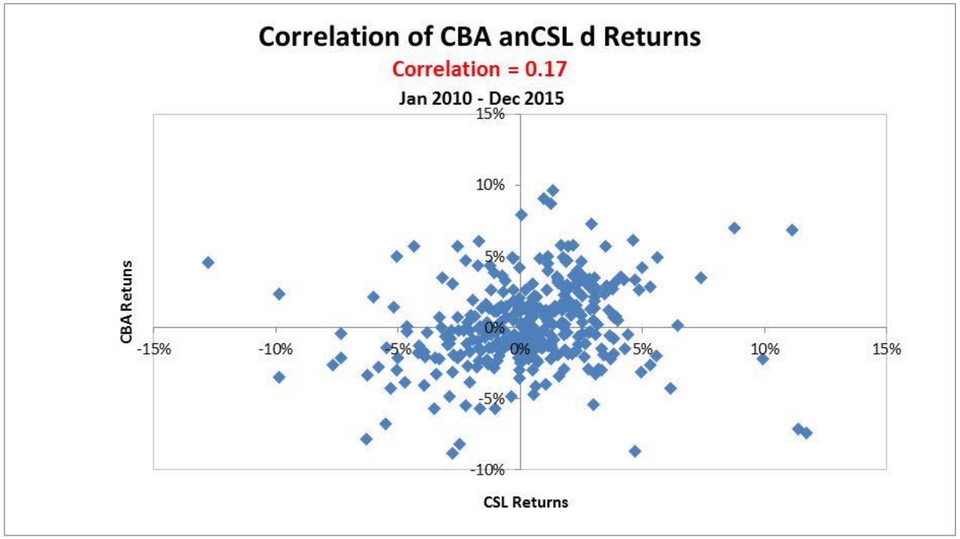
T-Bill

Low Variance/
Low Standard Deviation

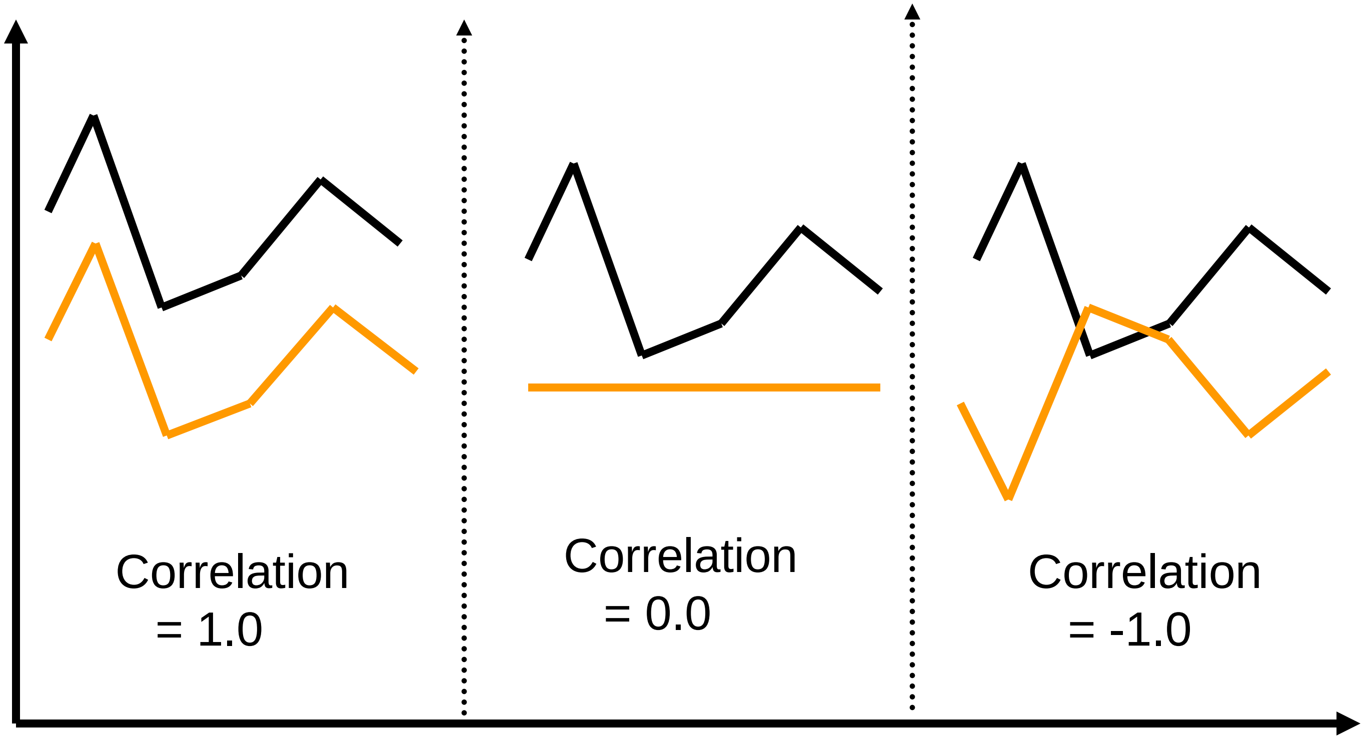
- The covariance coefficient measures the ***direction*** of the relationship between the returns on two different securities.

- The correlation coefficient measures the ***direction*** and the ***strength*** of the relationship between the returns on two different securities.





Interpreting Correlation

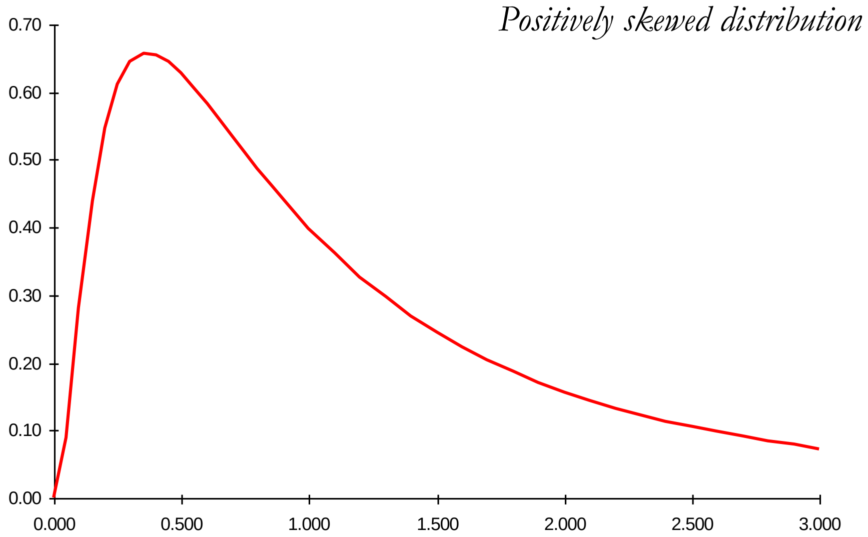


Skewness

- The skewness of a random variable X, denoted skew(X), measures the symmetry of a distribution about its mean.

- If X has a symmetric distribution then skew(X)=0 since positive and negative values in the formula for skewness cancel out.

- If skew(X) > 0 then the distribution of X has a long right tail.



- If skew(X) < 0 the distribution of X has a long left tail.

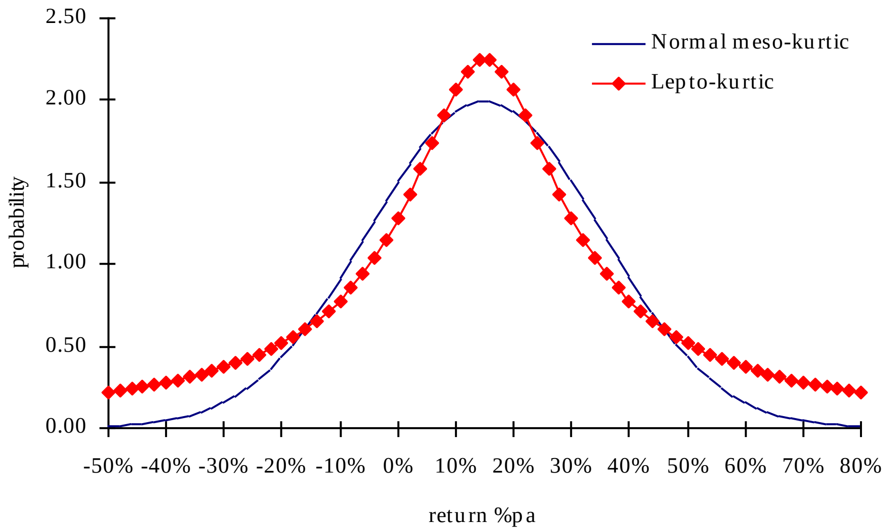
Kurtosis

- The kurtosis of a random variable X, denoted kurt(X), measures the thickness in the tails of a distribution.

- Leptokurtic distributions have greater probability of a extreme values compared to normal distribution.

- If the kurtosis is small then most of the observations are tightly clustered around the mean and there is very little probability of observing extreme values.

- EXCEL: =KURT(..)



Calculating the Statistics

Average of R

$$\bar{R} = \frac{\sum_{i=1}^{N} R_i}{N} = \frac{R_1 + R_2 + R_3 + R_4 + \cdots + R_N}{N}$$

Sample Variance of R

$$\sigma^2 = \frac{\sum_{i=1}^{N} \left(R_i - \bar{R}\right)^2}{N-1} = \frac{\left(R_1 - \bar{R}\right)^2 + \left(R_2 - \bar{R}\right)^2 + \cdots + \left(R_N - \bar{R}\right)^2}{N-1}$$

Standard Deviation of R

$$\sigma = \sqrt{\sigma^2\left(R\right)}$$

Calculating the Statistics

Sample Covariance between R^A and R^B

$$\sigma_{AB} = \frac{\sum_{i=1}^{N}\left(R_i^A - \bar{R}^A\right)\times\left(R_i^B - \bar{R}^B\right)}{N-1}$$

Correlation between R^A and R^B

$$\rho_{AB} = \frac{\text{Covariance between } R^A \text{ and } R^B}{\left(\text{Std.Dev.of } R^A\right)\times\left(\text{Std.Dev.of } R^B\right)}$$

$$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A \sigma_B}$$

Calculating the Statistics - alternatively

Measuring Expected Return $E(r)=\sum_i p_i r_i$

Variance and Standard Deviation of Returns

$$Variance = \sigma_i^2 = \sum_i p_i \left(r_i - E(r)\right)^2$$

$$\text{standard deviation} = \sqrt{\sigma_i^2}$$

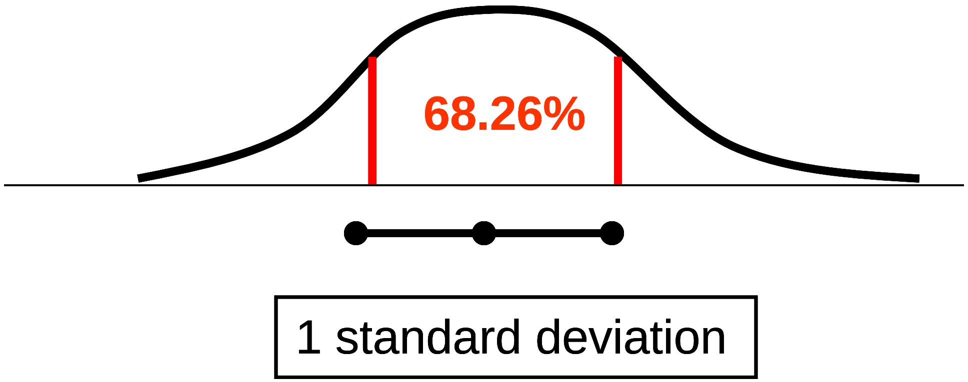
- **Covariance in returns, σ_{AB}, is defined as:**

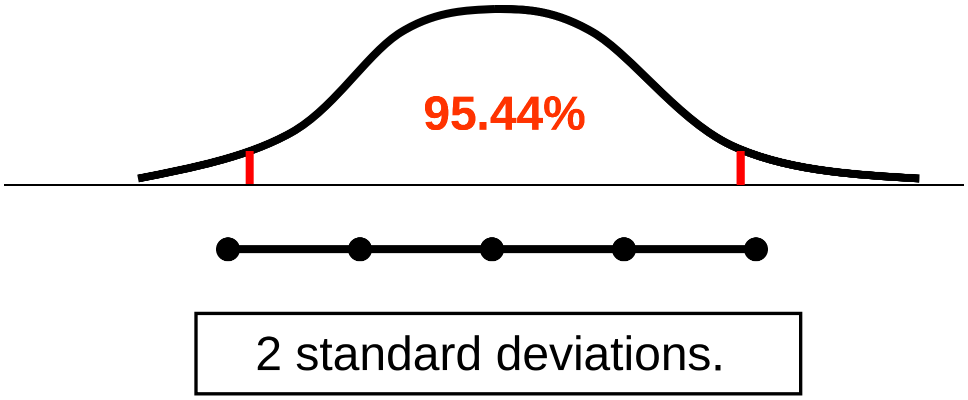
$$\sigma_{AB} = \sum_i p_i \left(r_{Ai} - E(r_A)\right)\left(r_{Bi} - E(r_B)\right) = \sum_i p_i r_{Ai} r_{Bi} - E(r_A)E(r_B)$$

The **correlation**, ρ_{AB}, is defined as: $\rho_{AB} = \dfrac{\sigma_{AB}}{\sigma_A \sigma_B}$

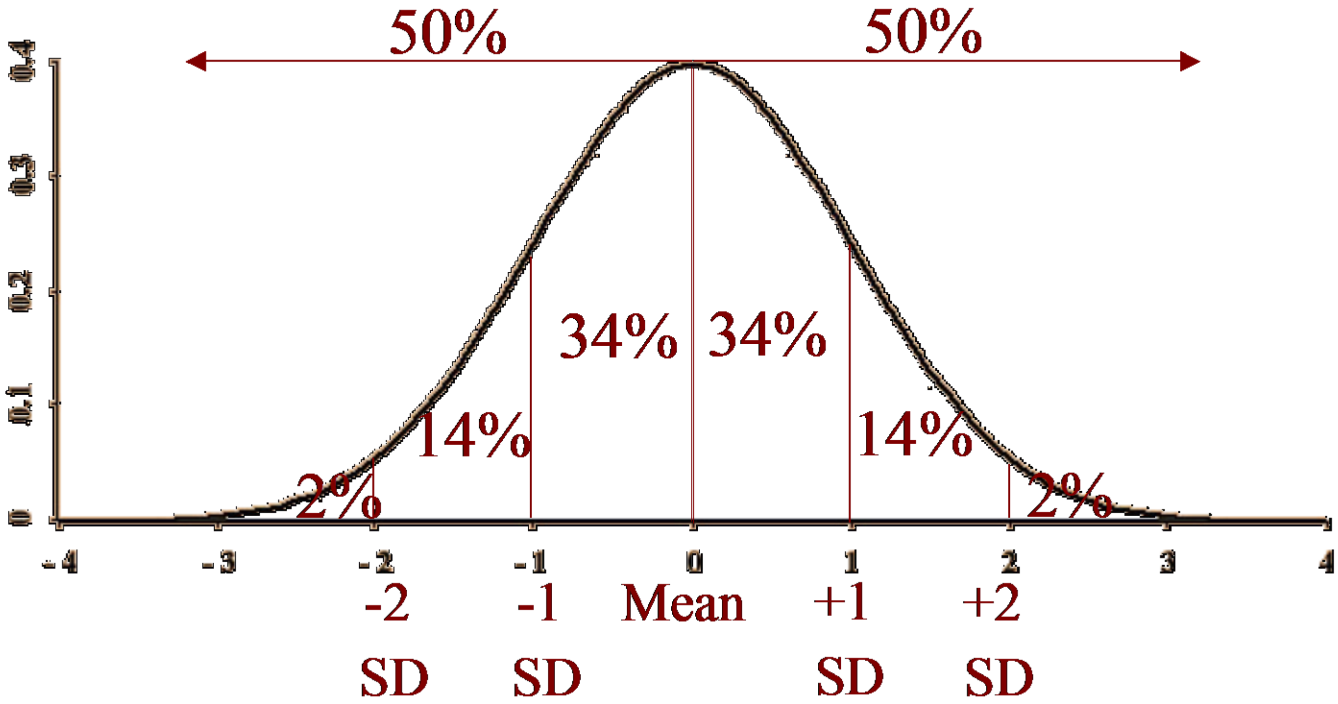
– ρ_{AB} is in the range [-1,1]

Interpretation of σ





Estimating Downside Risk



Interpretation of σ

- For Stock XYZ: Average Return = .03, σ = .11 (rounded)

- There is a 68% chance that stock XYZ falls between (.03 - .11) and (.03+.11), or -.08 and .14

- They say that there is a 95% chance that stock XYZ falls between (.03 – 2 x 0.11) and (.03 + 2 x 0.11), or -.19 and .25

Question 4. Statistical Calculations

Probability P	RIO Return (Monthly) R_{RIO}	NAB Return (Monthly) R_{NAB}
0.3	-10%	5%
0.4	5%	15%
0.2	10%	20%
0.1	15%	25%

a. Compute the mean returns for RIO and NAB.

b. Compute the standard deviations of returns for RIO and NAB.

c. Compute the covariance between returns for RIO and NAB.

d. Compute the correlation coefficient between the returns of RIO and NAB.

The Standard Normal Distribution

- The shape of the normal distribution is the familiar "bell curve".
- Often used in finance to describe the probabilistic behavior of stock returns .
- If a random variable X follows a *standard normal distribution X~N(0, 1).*
- This distribution is centered at zero and has a standard deviation of 1.
- The pdf of a normal random variable is given by

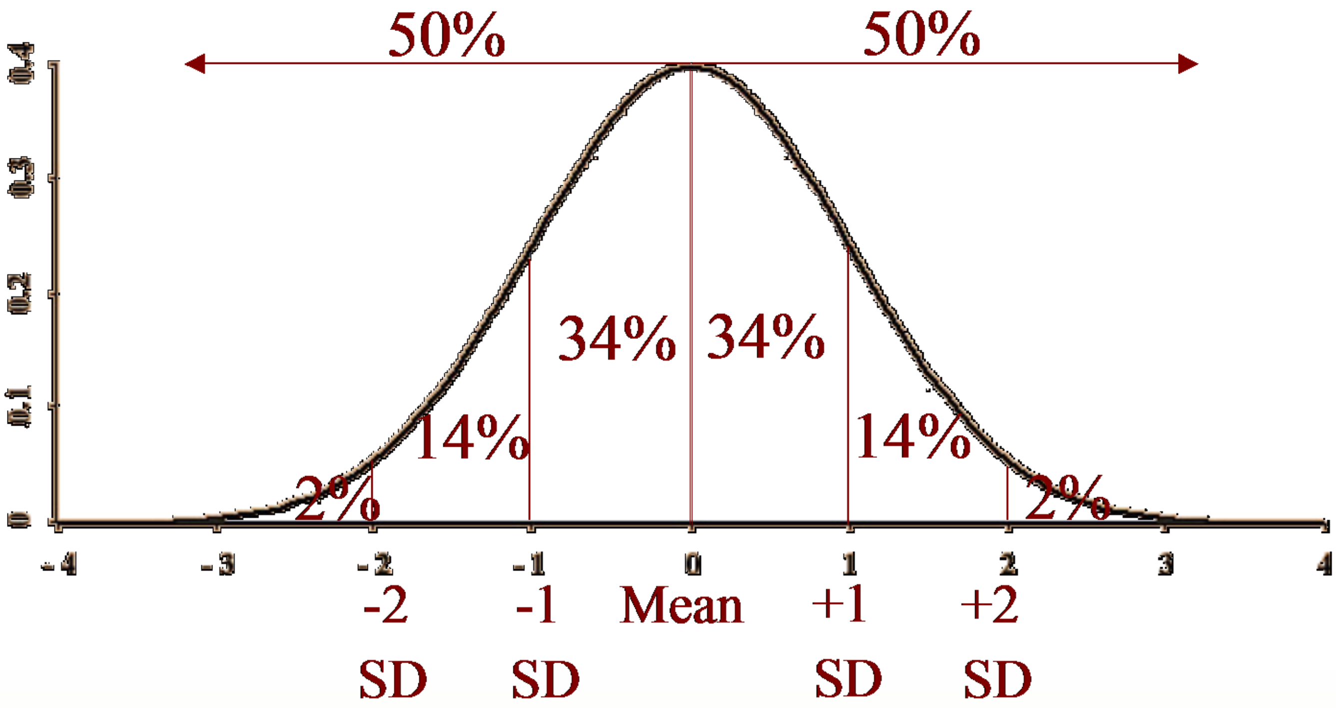
$$p(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}x^2} \quad -\infty \leq x \leq \infty$$

The Standard Normal Distribution……

- The area under the standard normal curve is one:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}.e^{-\frac{1}{2}x^2} dx = 1$$

- The distribution is symmetric about zero



The Standard Normal Distribution……

- The area under the normal curve

$$\Pr(a < X < b) = \int_a^b \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}x^2} dx$$

- Some useful results:

$$\Pr(-1 < X < 1) \approx 0.68$$

$$\Pr(-2 < X < 2) \approx 0.95$$

$$\Pr(-3 < X < 3) \approx 0.99$$

- **Finding areas under the normal curve**

$\Pr(X \leq z) = 1 - \Pr(X \geq z)$ and $\Pr(X \geq z) = 1 - \Pr(X \leq z)$

$$\Pr(X \geq z) = \Pr(X \leq -z)$$

$$\Pr(X \geq 0) = \Pr(X \leq 0) = 0.5$$

The Cumulative Distribution Function

- The cumulative distribution function (cdf), F, of a random variable X, (discrete or continuous) is the probability that X ≤ x

- The cdf has the following properties:

$$F(x) = \Pr(X \le x), \quad -\infty \le x \le \infty$$

(i) If $x_1 < x_2$ then $F(x_1) \le F(x_2)$

(ii) $F(-\infty) = 0$ and $F(\infty) = 1$

(iii) $\Pr(X > x) = 1 - F(x)$

(iv) $\Pr(x_1 < X \le x_2) = F(x_2) - F(x_1)$

Estimating Downside Risk

- Considers the risk that return will fall below some target value.
- Downside risk only measures the variability of returns below a target return.

Assume normal distribution for R.

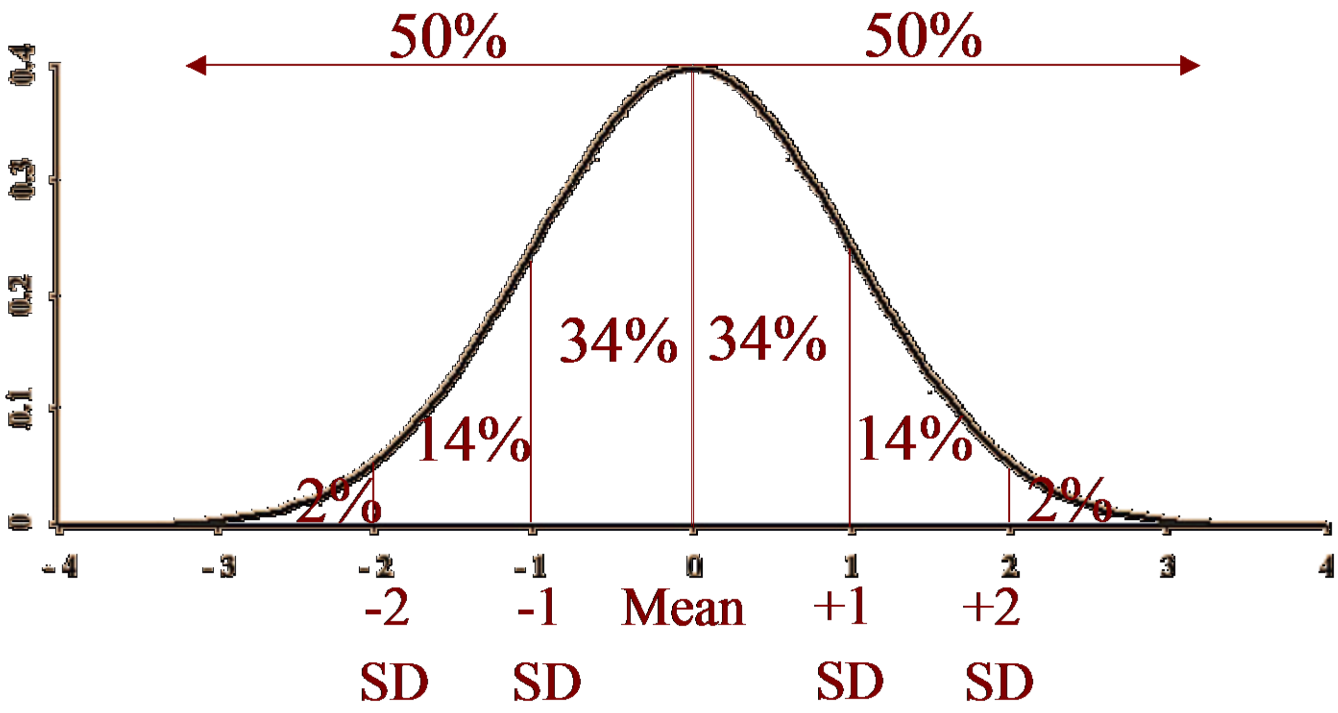
R ~ N (μ, σ^2) with mean = μ and standard deviation = σ

Specify a target return, R^*.

Evaluate investment strategy using the dispersion of returns below this target.

$$P(R < R^*) = P\left(\frac{R-\mu}{\sigma} < \frac{R^*-\mu}{\sigma}\right) = P\left(Z < \frac{R^*-\mu}{\sigma}\right)$$

Estimating Downside Risk



Downside Risk

Question 6.

- The annual returns of a portfolio have an expected return of 10% p.a and a variance of 0.0225 p.a.. Assume the returns are normally distributed.

a. What is the probability that returns are worse than −20%?

b. What is the probability that returns are negative?

Question 7.

- The annual returns of an emerging markets exchange-traded fund (ETF) have an expected return of 20.60% and a standard deviation of 30.85%. You are asked to estimate the likelihood of extreme return scenarios. Assume the returns are normally distributed.

- What is the probability that returns are worse than −30 percent?

Solutions

Time & Dollar Weighted Returns

Question 2.

Consider the following data. Assume that contributions are made at the beginning of the year.

Year	Year-End Value	Contributions
1	$22.0	-
2	$20.1	-
3	$32.0	$5.0

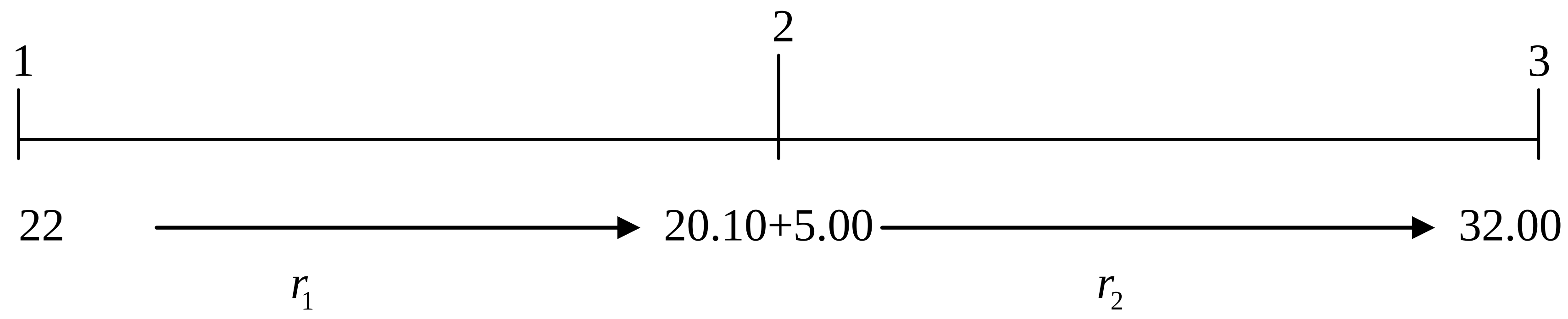
(a) What is the time-weighted rate of return on the fund?
(b) What is the value (dollar) weighted rate of return on the fund?

Answer:
(a) R_{tw} = 7.93%
(b) R_{vw} = 9.78%

Time & Dollar Weighted Returns

Question 2. Solution

	End Year 1	End Year 2	End Year 3
Value before Inflow		20.1	32.00
Inflow		5.0	
Value after inflow	22.0	25.1	32.00



Time & Dollar Weighted Returns

Question 2. Solution

$$r_1 = \frac{20.10 - 22.00}{22} = -0.08636$$

$$r_2 = \frac{32 - 25.1}{25.1} = 0.27490$$

$$\therefore r_{tw}: \quad [1 + r_{tw}]^2 = [1 + r_1][1 + r_2]$$

$$\therefore \quad r_{tw} = \{[1 + r_1][1 + r_2]\}^{1/2} - 1$$

$$r_{tw} = 7.93\%$$

Time & Dollar Weighted Returns

Question 2. Solution

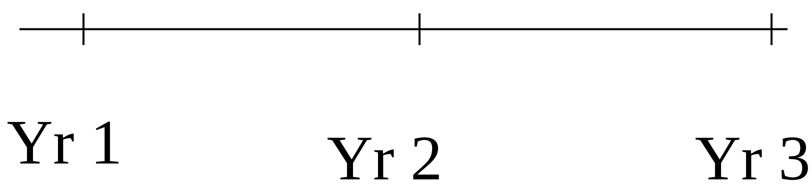
$$r_{vw}: \quad 22.0\left[1+r_{vw}\right]^2+5.0\left[1+r_{vw}\right]=32.00$$

Using IRR function in calculator:

r_{vw} = 9.78%

Question 3. **Solution**

Year-End Contribution	+6	+8	+7
Yr End Port. Value (After Yr-End Contribn)	106	148	167
Yr End Port. Value (Before Yr-End Contribn)	106 -6 =100	148 - 8 =*140*	167 - 7 =*160*
Begin of (next) Year Contribution	+5	+4	-
Total Amount Invested	106 + 5 = 111	148 + 4 =152	



Question 3. Solution

(a)

$$V_1 = 106 + 5 = 111$$

$$r_2 = \frac{148 - 8 - 111}{111} = 26.13\%$$

$$r_3 = \frac{167 - 7 - (148 + 4)}{148 + 4} = 5.26\%$$

Hence the time weighted return is

$$r = [(1.2613)(1.0526)]^{\frac{1}{2}} - 1 = 15.22\%$$

(b)

$$167 = 111(1 + r)^2 + 12(1 + r) + 7$$

$$111(1 + r)^2 + 12(1 + r) - 160 = 0$$

$$r = 14.8\%$$

Hence the dollar-weighted return is 14.8%.

Question 4. Solution

RIO			NAB	
Mean	0.02500		Mean	0.14000
Variance	0.00763		Variance	0.00440
Standard Deviation	0.08732		Standard Deviation	0.06633
	Covariance	0.00575		
	Correlation	0.99271		

Question 4. Solution

			R_{RIO} devn	P*[(R_{RIO} devn)^2]	R_{NAB} devn	P*[(R_{NAB} devn)^2]	(R_{RIO} devn) * (R_{NAB} devn)	P * [(R_{RIO} devn) * (R_{NAB} devn)]
	P * R_{RIO}	P * R_{NAB}	R_{RIO} - avg(R_{RIO})		R_{NAB} - avg(R_{NAB})			
	-0.03	0.015	-13%	0.00469	-9%	0.00243	1.13%	0.00338
	0.02	0.06	3%	0.00025	1%	4E-05	0.02%	0.00010
	0.02	0.04	8%	0.00113	6%	0.00072	0.45%	0.00090
	0.015	0.025	13%	0.00156	11%	0.00121	1.38%	0.00138
SUM	***0.025***	***0.14***		***0.007625***		***0.0044***		***0.00575***

Downside Risk

Question 6.

- The annual returns of a portfolio have an expected return of 10% p.a and a variance of 0.0225 p.a.. Assume the returns are normally distributed.

a. What is the probability that returns are worse than −20%?

b. What is the probability that returns are negative?

Downside Risk

Question 6. Solution

(a)

Step1. Determine the standard deviation:

$$\sqrt{\text{variance}} = \sqrt{.0225} = 15\%$$

Transform the random variable (Returns) and the Target return to standard normal variates:

$$\frac{R^* - \mu}{\sigma} = \frac{-.20 - .10}{0.15} = -2$$

$$\text{Note}: \mu = 10\%$$

So we are looking for P(Z<-2). ie the probability that Z is less than 2 sd below the mean on a standard normal distribution.

Using the standard normnal tables, we find that there is a 2.28% probability of this occuring.

Question 6. Solution

(b)

Step1. Determine the standard deviation:

$$\sqrt{\text{variance}} = \sqrt{.0225} = 15\%$$

Negative return : so set the target return $\theta = 0$.

Transform the random variables (Returns) and the Target return to standard normal variates:

$$\frac{R^{*} - \mu}{\sigma} = \frac{0 - .10}{0.15} = -\frac{2}{3}$$

$$P\left(Z < -\frac{2}{3}\right) = 37.07\%$$

- Question 7 : Solution

Transform to standard normal variate:

$$Z = \frac{R^* - \mu}{\sigma} = \frac{-.3 - .206}{.3085} = -1.64019$$

This is 1.64 sd below the mean on a standard normal distribution.
Using the standard normnal tables, we find that there is a
5% probability of this occuring.