

## The power and limitations of Monte Carlo simulations

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Explaining the past is much easier than predicting the future. This uncertainty raises a significant number of issues when creating an investment plan. Monte Carlo simulations will illuminate the nature of that uncertainty, but only if those using them understand how it should be applied – and its limitations.

The practical approach to creating the forecasted part of a plan has evolved over time. Estimates of future market returns were once based primarily on time value of money calculations. This approach is also known as deterministic modeling, whereby there is no randomness in the future outcome. For example, a plan would assume a long-term return on stocks of 10% for each year with no variability over time.

An alternative to – and improvement upon – deterministic models (like time value of money) is stochastic models (such as Monte Carlo simulations) that incorporate randomness into the modeling process. The use of Monte Carlo tools has increased considerably over the last decade which can be attributed to lower computing costs, increased recognition that returns are random, and the need to provide more robust plans to investors.

In most Monte Carlo tools, returns and inflation are treated as random and they vary based on an assumed mean, standard deviation and correlation. Those inputs are defined by the user and have a considerable impact on the results of any simulation. For example, if you were to set the standard deviations to zero in these types of models, you would effectively run a deterministic simulation, because each return would be assumed to be known with absolute certainty and there would be no assumed variability in the forecast.

### USER ERROR

Monte Carlo simulation has received a lot of criticism, though not always for valid reasons.

One common criticism is that such tools may not incorporate the "fat tailed" nature of return distributions, as well as things like autocorrelation (when returns of a variable, like inflation, are correlated over time). But this argument is like saying all cars are slow. There are no constraints to Monte Carlo simulation, only constraints that users create in a model (or constraints that users are forced to deal with when using someone else's model). Non-normal asset-class returns and autocorrelations can be incorporated into Monte Carlo

simulations, albeit with proper care. Like any model, you need quality inputs to get quality outputs.

Nonetheless, most Monte Carlo investment-planning software is based on a normal distribution for returns, with inputs including expected asset-class returns, standard deviations and correlations. Outcomes at the tails may not be precise, because we cannot be certain about both the accuracy of our inputs and the underlying distribution for the returns. Those probabilities of success or failure for a plan calculated through Monte Carlo simulation are only estimates.

However, if the calculated success rate is low, say 15% to 25%, we know that a plan generating these numbers is in danger and that the plan should be immediately reconsidered. Likewise, if the calculated probability of success is high, say 80% or greater, we know that the plan is on the right track and that the investor can proceed as indicated. No plan is set-it-and-forget-it, and frequent revisits to any plan are desirable. But Monte Carlo output in this range is an indication that based on the best current assessment, the investor can be reasonably confident about the plan while understanding that future tweaks may still be needed as events unfold.

## RETURN DISTRIBUTIONS AND VARIANCE DRAIN

Monte Carlo tools are primarily used to show the potential impact of random returns over time. For example, if we assume the annual returns on stocks follow a normal return distribution (which isn't too far of a stretch, as things become less normal over smaller time intervals) with an average return of 10% and a standard deviation of 20%, we are saying there is approximately a 68% chance the return in a given year will be between -10% and +30%, and there is approximately a 95% chance the return will be between -30% and +50%.

Therefore, we are recognising the relative uncertainty associated with future returns. This uncertainty and the shape of the return distribution have important impacts on the wealth accumulated by an investor. First, since returns are random (both positive and negative), the actual account growth will be lower than the expected return, due to variance drain (also called volatility drag). A simple example to demonstrate the effect of variance drain is if you have a portfolio that returns +100% in the first year and -50% in the following year. The average return is +25%  $[(+100\% - 50\%)/2 = +25\%]$ , but the compounded return is 0% (\$1 would grow to \$2 at the end of the first year, but would subsequently decrease to \$1 by the end of the second year). Therefore, with variance drain, negative returns have an outsized impact on realised returns.

The realised return of an investor that takes into account variance drain is often referred to as the geometric or compound return. The geometric return can be estimated by subtracting half the variance from the arithmetic return. Assuming an expected return of 10% and a standard deviation of 20%, the geometric return would be 8%  $(10\% - ((20\%)^2)/2)$  and the

variance drain would be 2%. The geometric return should be used when running a deterministic (time value of money) calculation, as it better reflects the expected realised return of the investor.

## RETURN CONSIDERATIONS AND ALTERNATIVE APPROACHES

A problem with Monte Carlo tools that is exacerbated today is that they can often paint an unrealistic picture of returns. For example, an average expected return for cash of 2% is unrealistic (since the return on cash today is closer to 0%). This carries important implications for sequence risk for retirees, because the order returns are incurred has a significant impact on the potential success of a given strategy. This is a concept we have written a number of papers on, using a variety of Monte Carlo models.<sup>1</sup>

One method to incorporate information about today's return environment (e.g., the yield on bonds) is to use an autoregressive model. An autoregressive model captures a relationship between the previous value and the next value. This is important when projecting things like bond yields versus stocks, because bond returns (and interest rates) are auto-correlated. However there are other metrics (e.g., dividend yields and Shiller's Cyclically Adjusted Price-to-Earnings ratio) that have historically been useful when projecting the future return on equities.

There are alternative ways to incorporate returns into a Monte Carlo tool. One method is to use historical series of returns (e.g., the returns for the US stock market over a long time period). One significant problem with this approach is that it assumes what has happened historically will happen in the future. It implicitly incorporates the recurrence of historical events and allows for only a limited amount of data. Two ways to make using historical series of returns more attractive are to reduce returns to more conservative levels and/or use international data, when available.

## EXTENDING THE RANDOMNESS

While most investment Monte Carlo tools treat returns as random, there are other variables that are random as well. One of the most important is life expectancy. The most common method used to forecast retirement is to assume that the portfolio must provide income for some fixed period, such as 30 years, and the resulting probability of success provides information about the relative safety of the strategy.

The problem with this approach is that it doesn't convey the true level of success, because success isn't defined appropriately.

In reality, two things have to happen for individuals to have "unsuccessful" retirements. They have to still be alive and they have to run out of money. When modeling this type of outcome, you have two random variables – life expectancy and returns. To model this, you

must allow for the probability of an individual (or one member of a couple) passing away in a given year, but the individual must still be alive and the portfolio must be depleted for failure to occur. This approach can result in considerable differences in the estimate of what the safe initial withdrawal rate should be from a retirement portfolio when compared to a model that does not incorporate variable mortality.

## CONTRASTING DETERMINISTIC AND STOCHASTIC SIMULATIONS

Monte Carlo is an incredibly powerful way to model the randomness of life. It provides a colorful perspective of the possible outcomes versus the very black-and-white (i.e., binary) nature of a deterministic projection. By definition, a deterministic projection provides the user with a 50% probability of success based on the assumed return. If the assumed returns are arithmetic (and not geometric, as they should be), the actual probability of success is likely less than 50%.

The fact that a time value of money (i.e. deterministic) calculation results in approximately a 50% probability of success has important implications. What appears to be certain from an investor's perspective (because you may have provided no other comparisons) is considerably less certain.

We are not suggesting that practitioners tell investors they have an 82.15% probability of success, since that probability suggests a level of certainty that is highly specific based on the parameters used in the model. There will be estimation error, because we don't know what's going to happen in the future and therefore there will be errors in our estimates. An 82.15% probability of success is reasonably high, and that's what should be said when attempting to convey to an investor the relative certainty of a strategy.

## CONCLUSIONS

There are no limitations when running a Monte Carlo simulation, although you may be limited based on the tools you have available, or your ability to build such tools yourself. In our next piece, we'll focus on the importance of assumptions, particularly return assumptions, when running a Monte Carlo simulation.

## ENDNOTE

1. As an example, see [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=2286146](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2286146).



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